Exercise 55

Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(t) = t - \sqrt[3]{t}, \quad [-1, 4]$$

Solution

Take the derivative of the function.

$$\begin{aligned} f'(t) &= \frac{d}{dt}\left(t - \sqrt[3]{t}\right) \\ &= 1 - \frac{1}{3}t^{-2/3} \\ &= 1 - \frac{1}{3t^{2/3}} \\ &= \frac{3t^{2/3} - 1}{3t^{2/3}} \end{aligned}$$

Set what's in the numerator equal to zero, and set what's in the denominator equal to zero. Solve both equations for t.

$$\begin{aligned} 3t^{2/3} - 1 &= 0 & \qquad 3t^{2/3} &= 0 \\ t^{2/3} &= \frac{1}{3} & t^{2/3} &= 0 \\ t^2 &= \frac{1}{27} & t &= 0 \\ t = -\frac{1}{\sqrt{27}} \quad \text{or} \quad t &= \frac{1}{\sqrt{27}} & t &= 0 \end{aligned}$$

$t = -1/\sqrt{27}$ and $t = 0$ and $t = 1/\sqrt{27}$ are within $[-1, 4]$, so evaluate f at these values.

$$f\left(-\frac{1}{\sqrt{27}}\right) = -\frac{1}{\sqrt{27}} - \sqrt[3]{-\frac{1}{\sqrt{27}}} = \frac{2}{3\sqrt{3}} \approx 0.3849$$

$$f(0) = 0 - \sqrt[3]{0} = 0$$

$$f\left(\frac{1}{\sqrt{27}}\right) = \frac{1}{\sqrt{27}} - \sqrt[3]{\frac{1}{\sqrt{27}}} = -\frac{2}{3\sqrt{3}} \approx -0.3849 \qquad \text{(absolute minimum)}$$

Now evaluate the function at the endpoints of the interval.

$$f(-1) = -1 - \sqrt[3]{-1} = 0$$

$$f(4) = 4 - \sqrt[3]{4} = 4 - 2^{2/3} \approx 2.4126 \qquad \text{(absolute maximum)}$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $[-1, 4]$.

The graph of the function below illustrates these results.